Visuospatial representation and arithmetical thinking in students with Special Educational Needs

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Dyslexia Guild Annual Conference © June 2017

AIMS

Investigate the difficulties of low-attaining secondary-age students with multiplicative thinking, via their representational strategies for arithmetical problems.

Explore the use of nonstandard visuospatial representations in tuition, for improving struggling students' understanding of multiplication and division.

Develop methods for qualitative diagnosis and assessment of arithmetical-representational strengths, weaknesses and microprogressions in students' work.

PARTICIPANTS

Characteristics of research participants:

- aged 11-16
- in mainstream schools, placed in ‘bottom sets’
- particularly low-attaining and/or with SEND
- understood addition/subtraction, and could carry out some tasks with additive structure
- had some experience with multiplication/division – and negative feelings towards them
- could recall some multiplication facts

TASKS

Characteristics of tasks used:

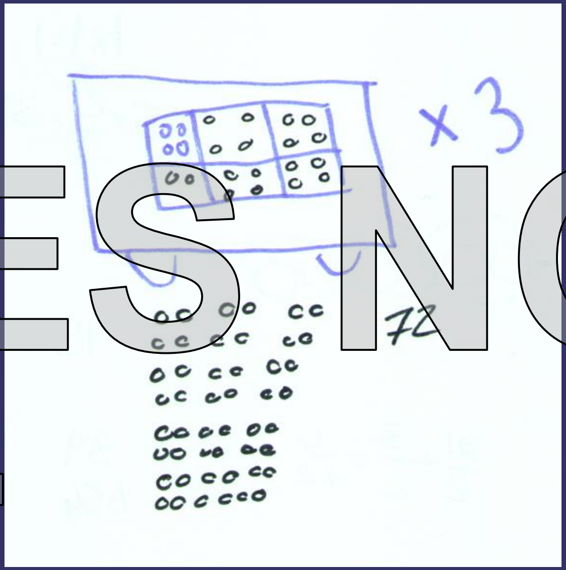
- natural numbers (max 3 digits)
- partitive and quotitive division
- multiplication (process and structure)
- scenario (e.g. biscuits, buses, clothing combinations)
- bare (e.g. number symbols, 2D and 3D arrays)
- psychologically commutative and non-commutative

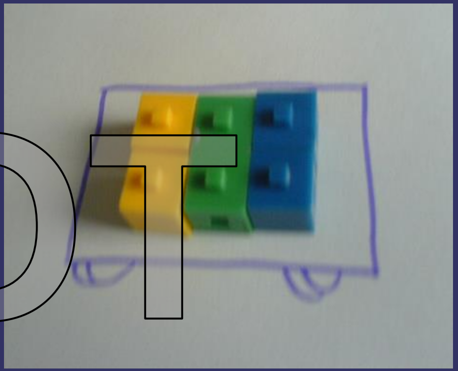
REPRESENTATIONAL MEDIA

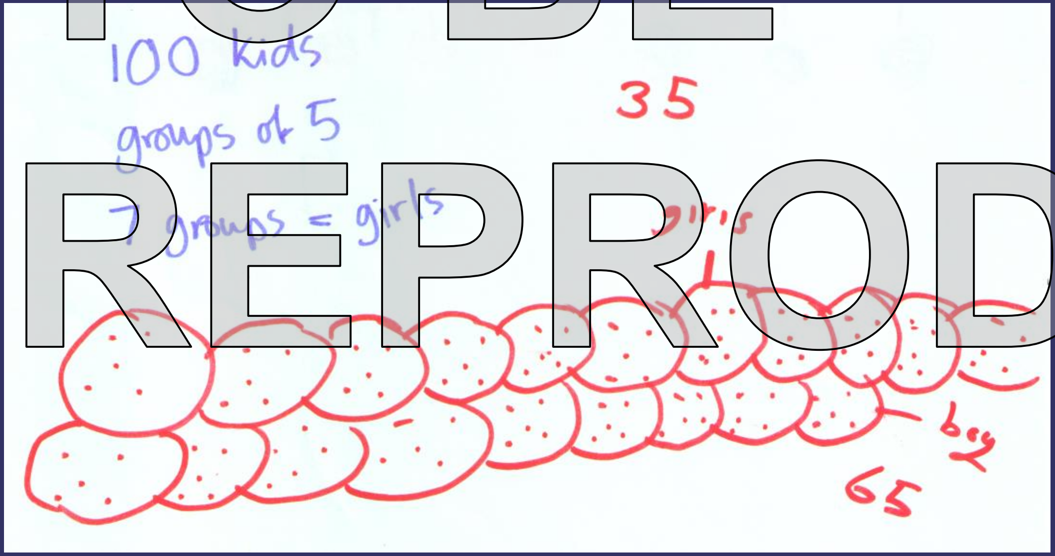
The representational media available to participants were:

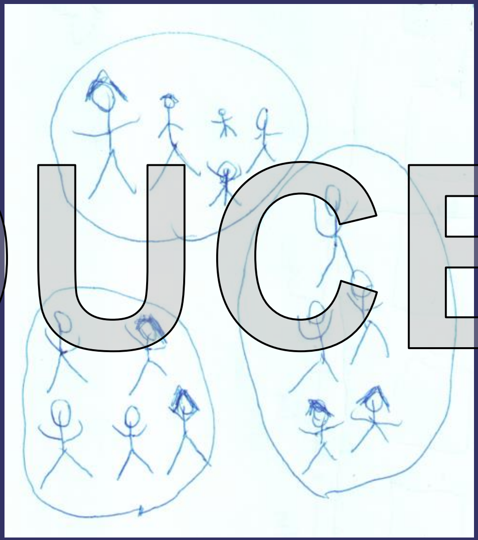
- paper and coloured felt tip pens
- coloured multilink cubes
- small bowls
- cut lengths of drinking straws
- own hands
- (internal)

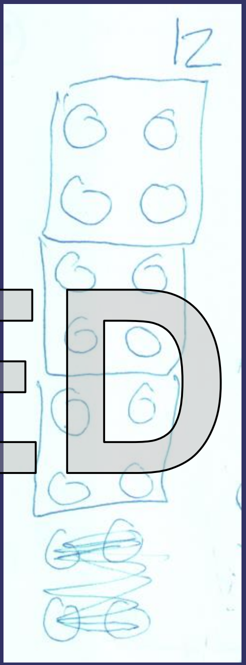
SAMPLE REPRESENTATIONS: INITIAL











IMAGES NOT TO BE REPRODUCED

TASK: HOLIDAY CLOTHES

A person has **6** t-shirts and **4** pairs of trousers in their suitcase.

The t-shirts are: **white**, **blue**, **green**, **brown**, **red** and **yellow**.

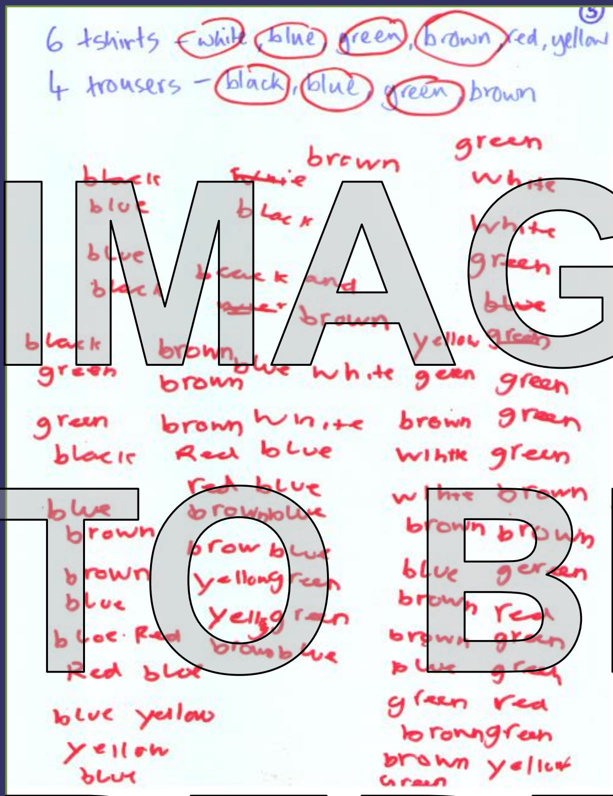
The trousers are: **black**, **blue**, **green** and **brown**.

How many different possible outfits can be made?

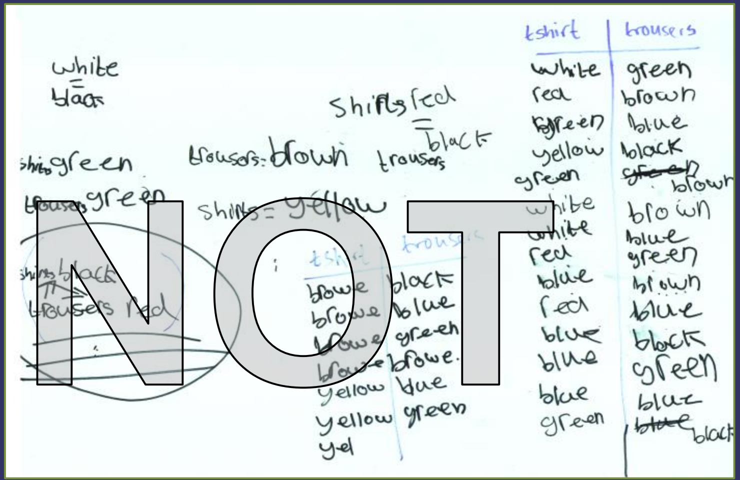
(Example outfit: blue trousers with white t-shirt)

- *What is the answer?*
- *How did you work it out?*
- *Why might a student get stuck?*
- *What might help them?*

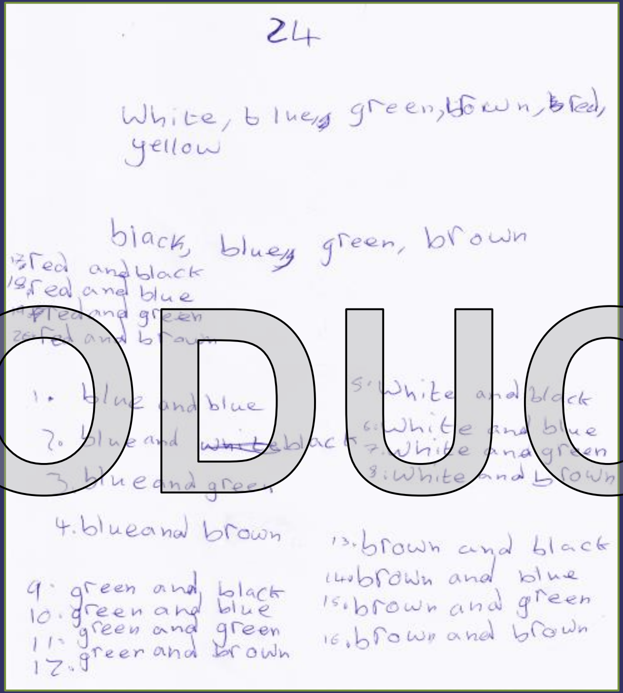
HOLIDAY CLOTHES – WRITTEN RESPONSES



Jenny, Y7



Tasha, Y8



Danny, Y8

all images from Finesilver (2009/2014)

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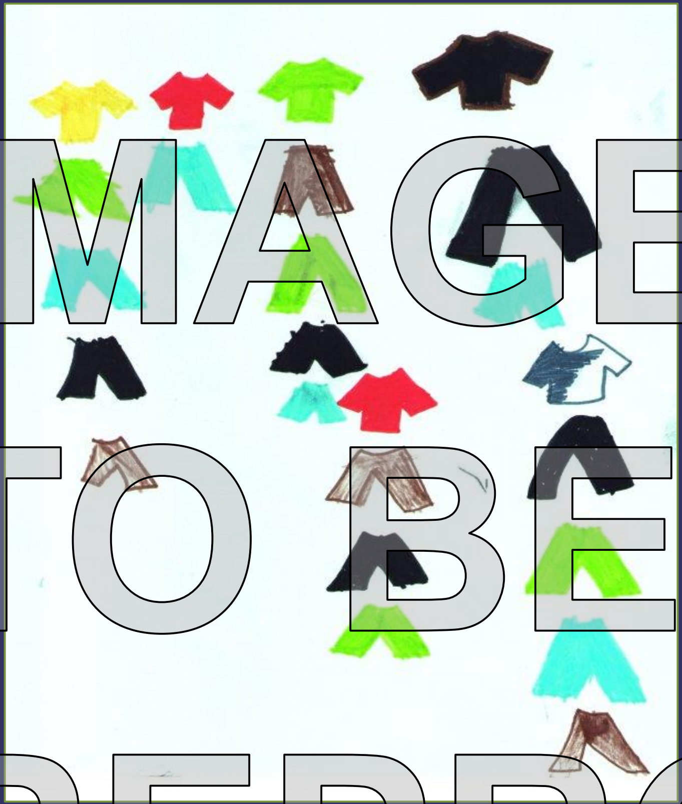
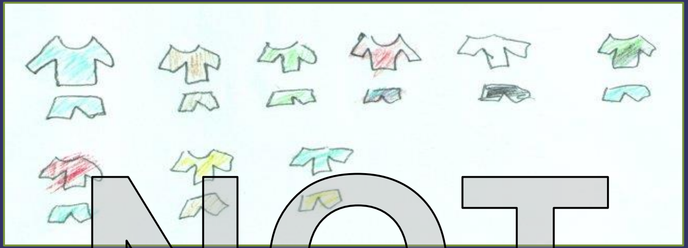
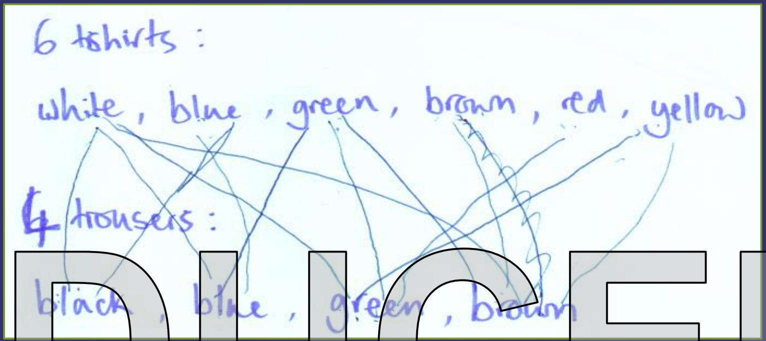
HOLIDAY CLOTHES – DRAWN RESPONSES



Kieran, Y7



Oscar, Y9
(5 mins)



George, Y8

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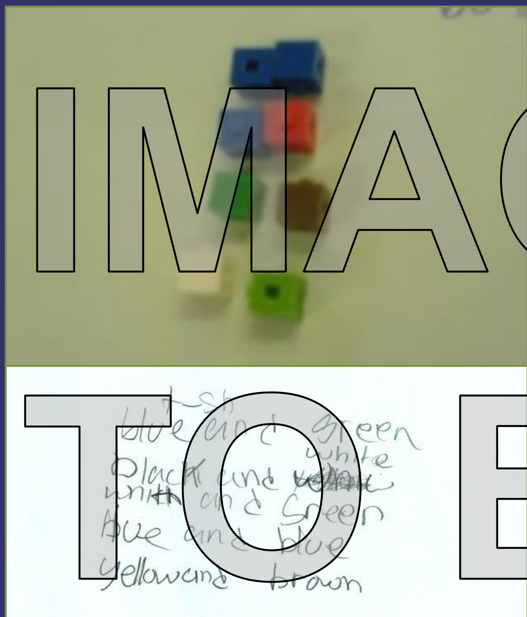
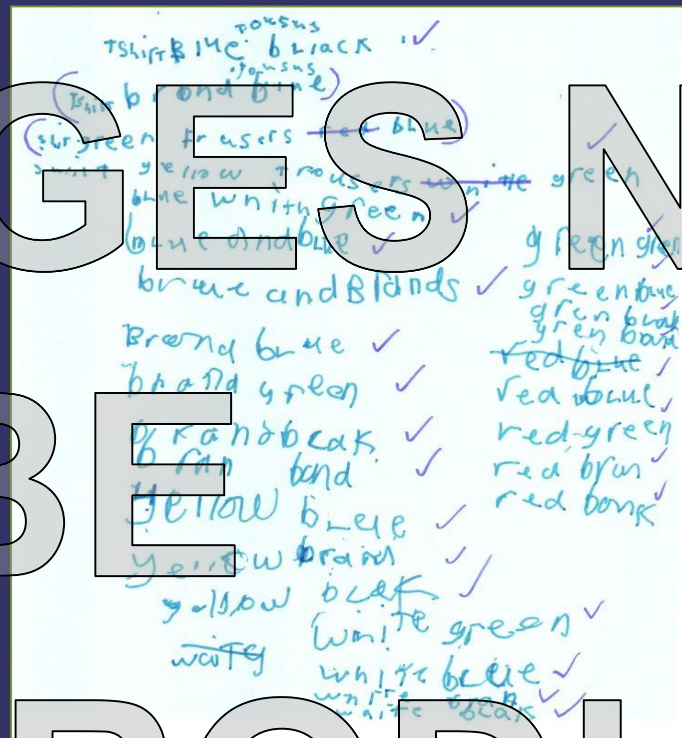
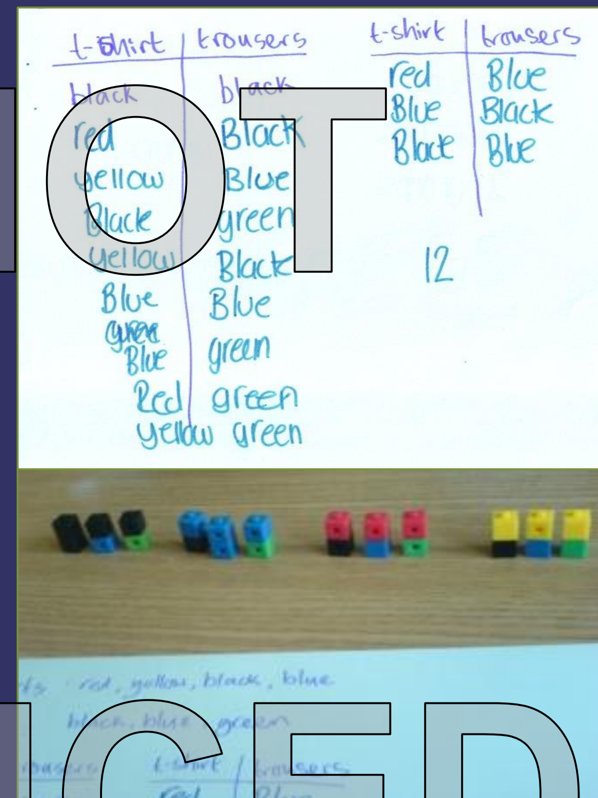
HOLIDAY CLOTHES – MODELLED RESPONSES



Sidney, Y9
(5 mins)



Harvey, Y7



Paula, Y10

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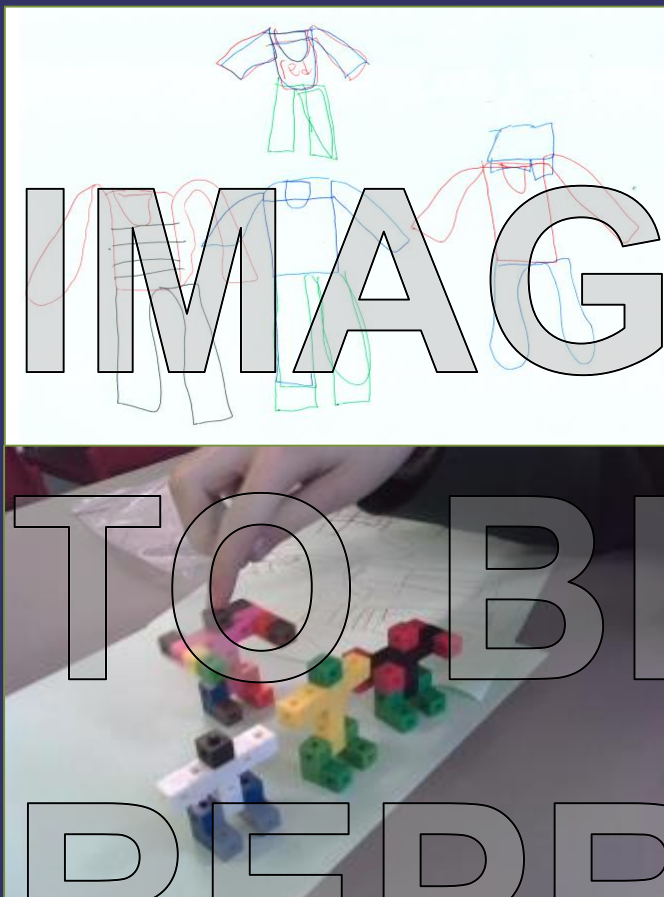
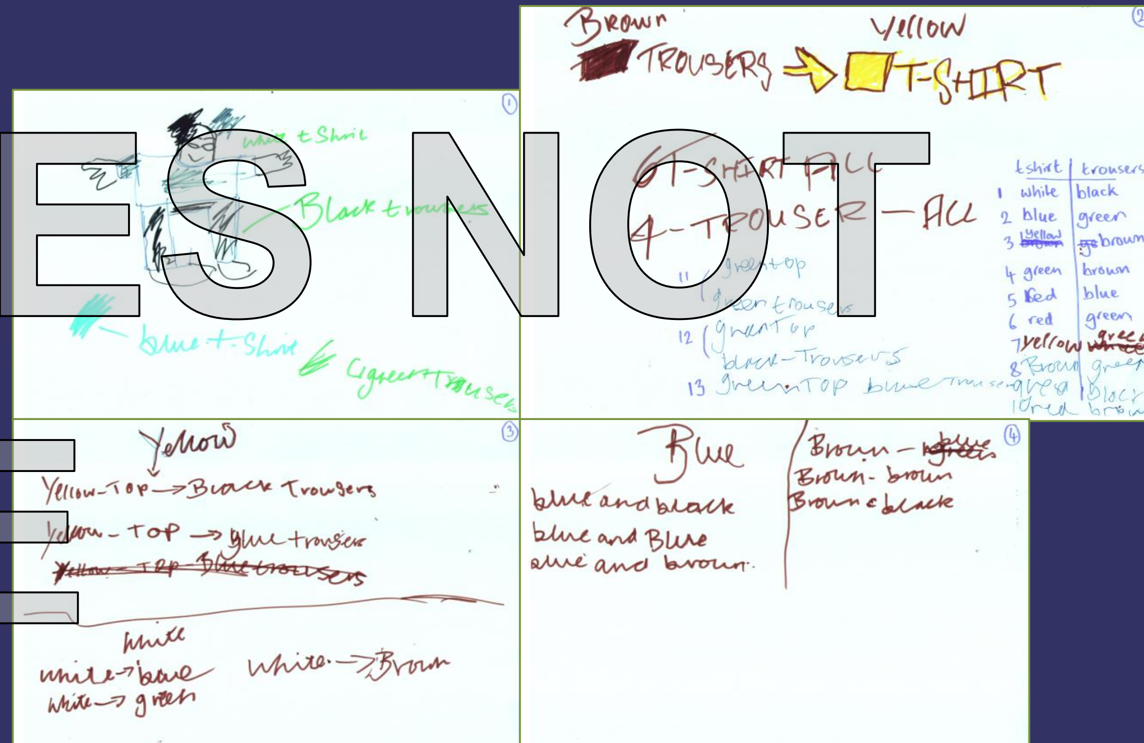
HOLIDAY CLOTHES – MIXED-MODE RESPONSES



Leo, Y7



Vince, Y7

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all images from Finesilver (2009/2014)

HOLIDAY CLOTHES – INTERPRETATION

Mode e.g. modelling, drawing, writing, symbols, gesture

Media preferred materials, e.g. cubes, pen/paper, fingers

Motion e.g. static once created, or involving ongoing movement of elements

Resemblance between the drawing or model and the task scenario described (NB This is related to the morecommonly-used ‘abstraction’.)

Consistency i.e. whether a single representational strategy was used from start to finish, or changes occurred

Completeness i.e. whether the external representation had to be ‘finished’ for solution

MATHEMATICAL 'ABILITY'

"[C]hildren's knowledge of mathematics is extraordinarily complex and often much different from what we had supposed it to be . . .

In the case of every child we have interviewed or observed, there have emerged startling contradictions, unsuspected strengths or weaknesses, and fascinating complexities."

Ginsburg (1972), in Dowker (2005) *Individual differences in arithmetic*

DIFFERENCES IN MATHEMATICAL ABILITIES

- Mathematical 'ability' is made up of many different components and subcomponents.
- There can be significant variation in how difficult a single individual, or individuals supposedly at the same 'level' find different subcomponents.
- Differences are due to specific patterns of strength and weakness, which may relate to SEN/D characteristics.
- Levels of actual performance on any occasion are influenced by factors which may be:
  - Internal to the individual (cognitive, affective)
  - External (environmental, pedagogical)

WORD PROBLEMS

"[Word problems are] the castor oil of the mathematics curriculum: fairly unpleasant but possibly good for you"

(Askew, 2003)

- *Is this fair?*

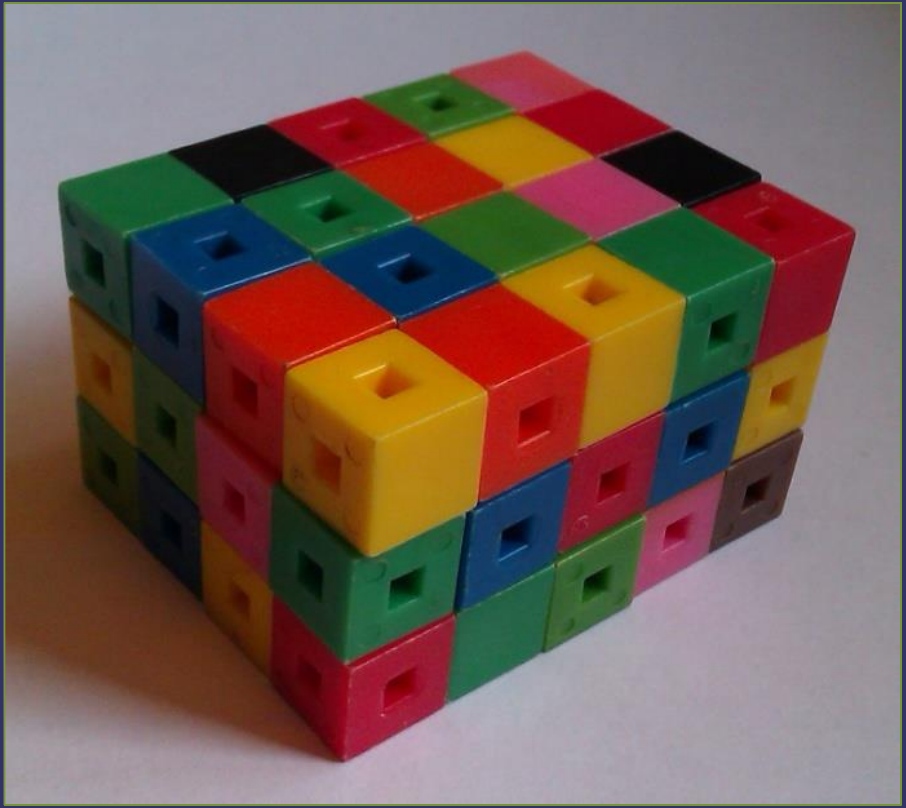
It depends on various factors:

- the learner(s) – abilities, attitudes, experiences
- language used
- representational modes & media available
- **imaginability of scenario**

(Item removed for copyright reasons)

TASK: CUBOID

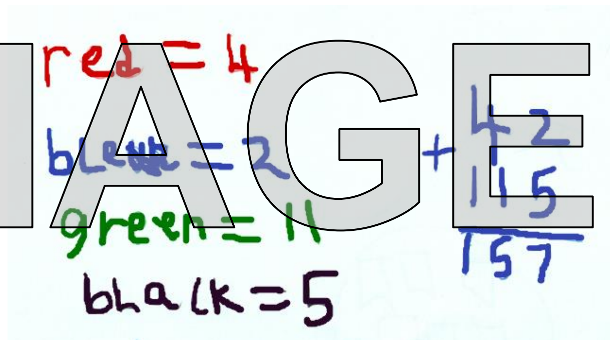
How many of the unit cubes make up the block?



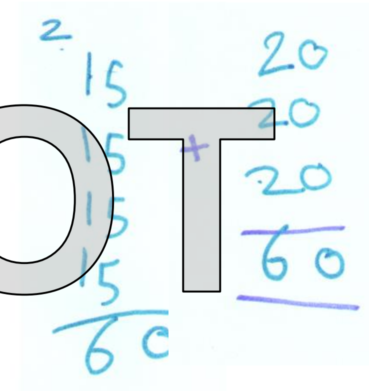
- *What is the answer?*
- *How did you work it out?*
- *Why might a student get stuck?*
- *What might help them?*

CUBOID – SELECTED RESPONSES

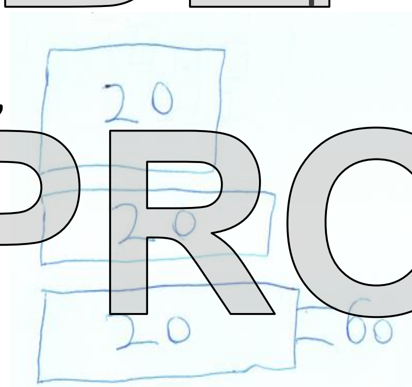
Leo (Y7) – first attempt



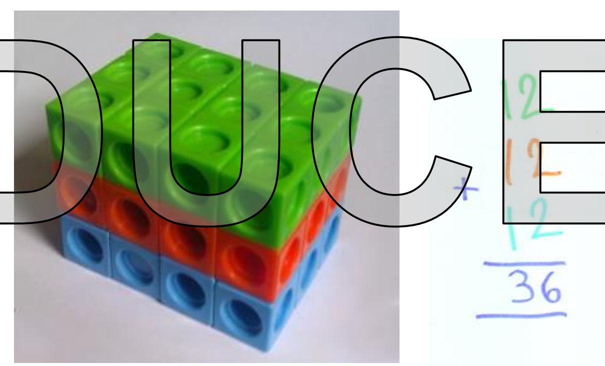
Tasha (Y8)
three
different
orientations



Leo (Y7) – second attempt
'pulling out
the drawers'



Paula (Y10)
3-colour block, 3-colour sum



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CUBOID – INTERPRETATION

Enumeration: Do they work out the total number of cubes by…?

- Multiplication
- Addition/Step-counting
- Counting
  - Step counting
  - Rhythmic counting
  - Grouped counting
  - Unitary counting

Spatial structuring: Do they conceptualise the cubes as...?

- 3D multiplicative structure
- stacked set of 2D layers
- arrayed set of 1D columns
- set of faces (surface area)

Errors: If they go wrong, is it…?

- Spatial structuring
- Numeric calculation
- Fact retrieval
- Verbal count sequence
- Visuospatial/kinaesthetic

ARITHMETICAL THINKING

Some of the major skills called upon:

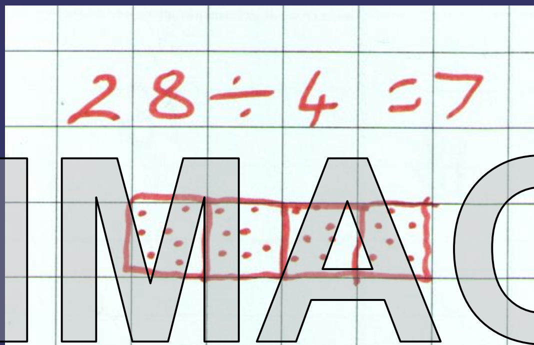
- Comparison and estimation of quantities
- Interpretation and manipulation of arbitrary symbols
- Memory (verbal & visuospatial)
- Sequencing
- Pattern recognition
- Task-based strategizing
- …

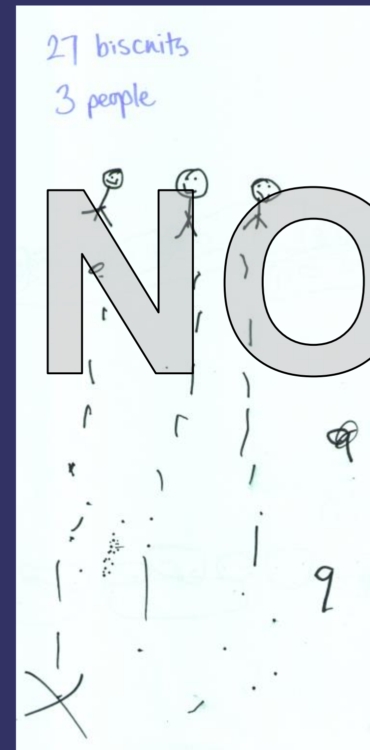
Learners with SEN/D need to be metastrategic – not rely on skills which are (currently) weak, and make use of strengths.

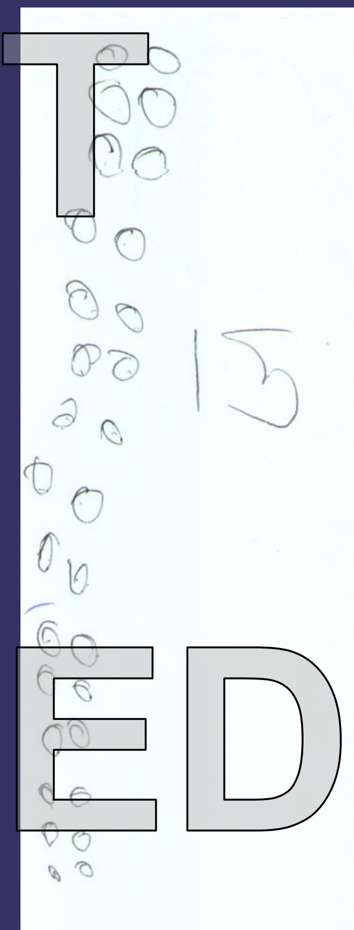
POTENTIAL BARRIERS

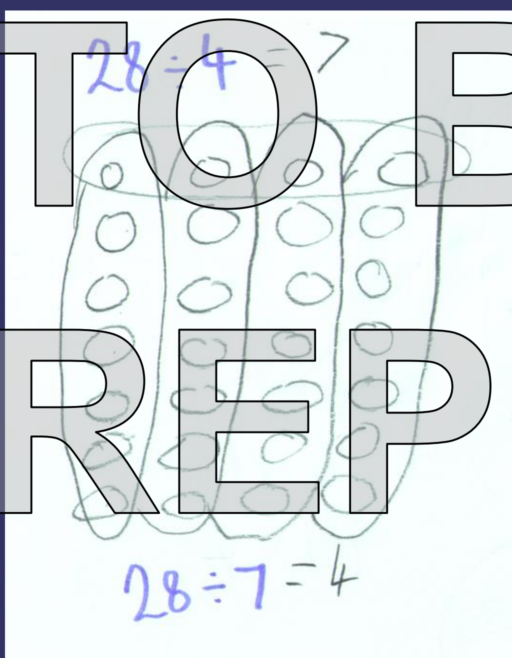
- No ‘clean slates’ – standard teaching approaches have been tried (maybe many times)
- Fear and loathing attached to operations, from past failure – avoidance
- Shame in admitting to lack of understanding/capability – concealment of ‘immature’ strategies
- Belief that there is only a single acceptable method for a given task type (from teachers and peer pressure)
- *Sunk cost bias* – not wanting previous hard work to ‘go to waste’, so persisting with unsuccessful strategies

SAMPLE REPRESENTATIONS: 'BISCUITS'









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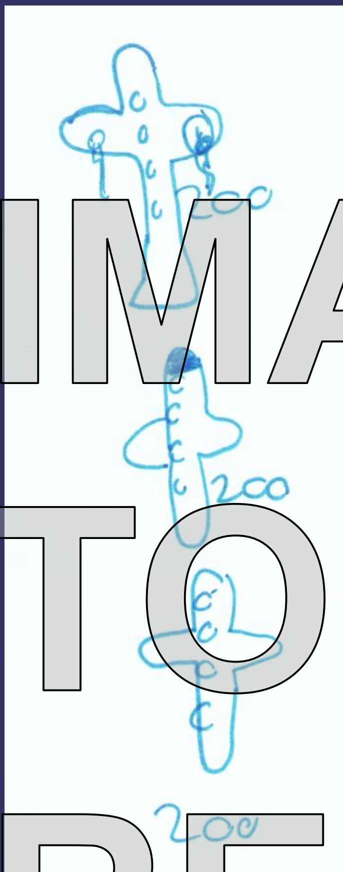
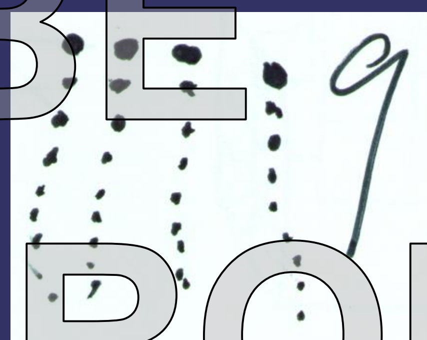
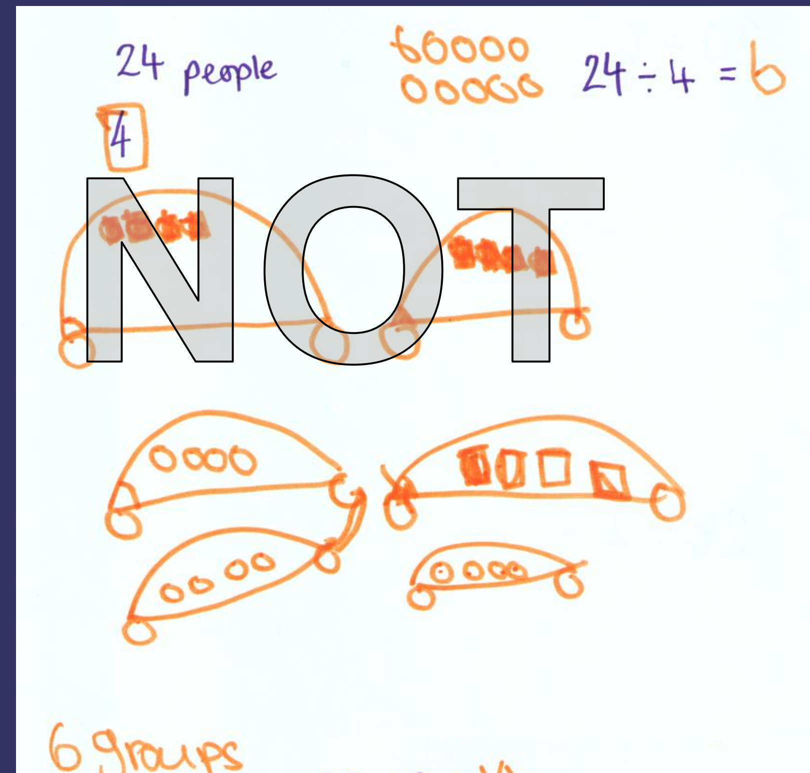
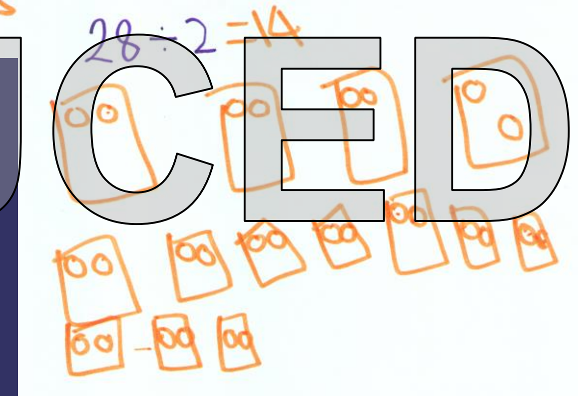
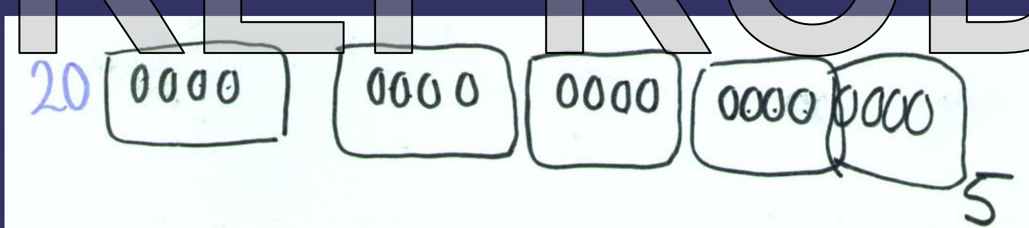
SAMPLE REPRESENTATIONS: 'PASSENGERS'











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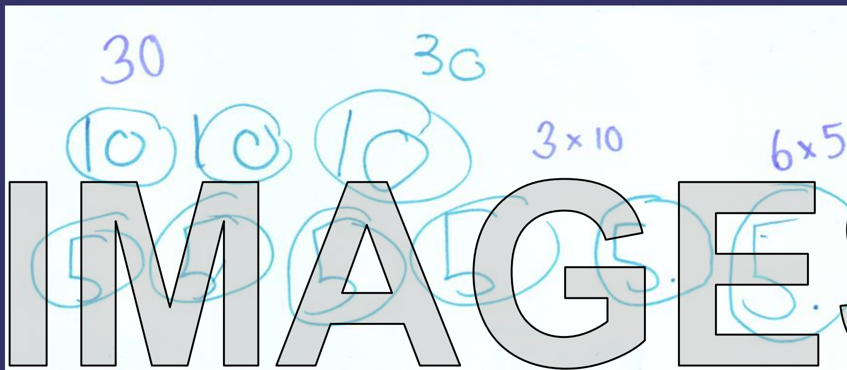
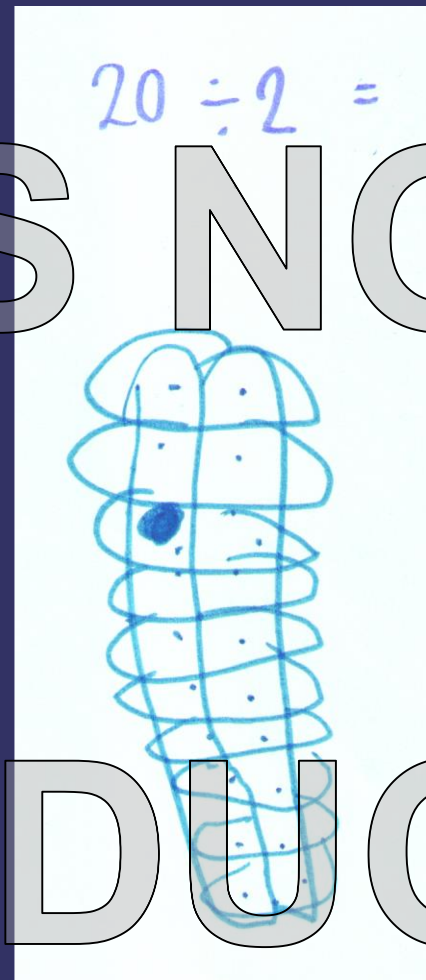
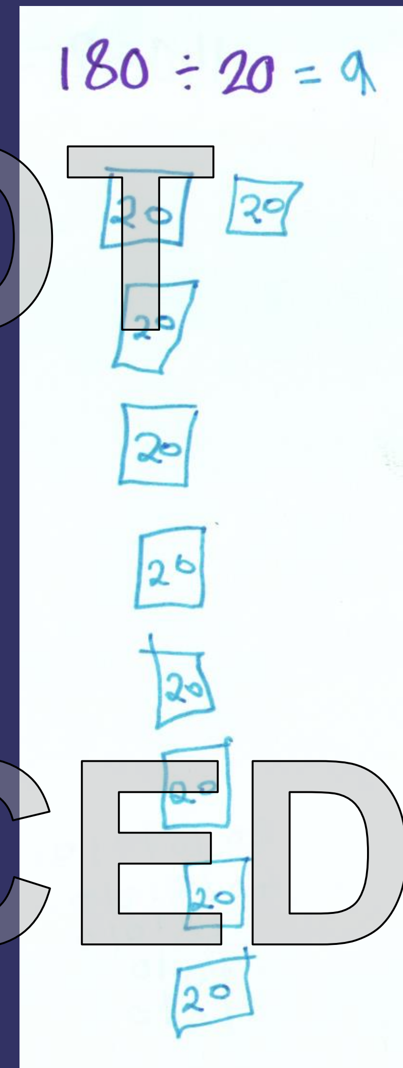
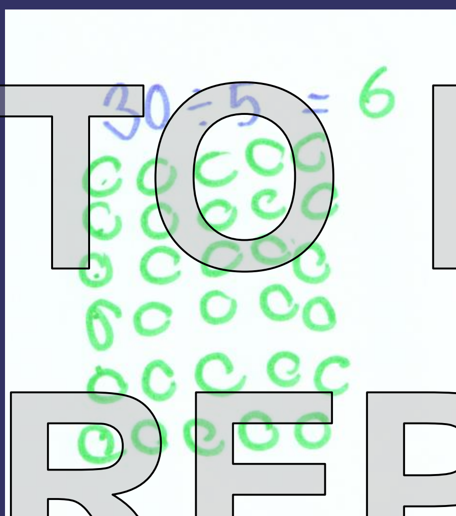
SAMPLE REPRESENTATIONS: BARE TASKS









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BASIC QUALITATIVE ANALYTICAL FRAMEWORK

Types of representations created:

Media e.g. cubes, pen/paper

Mode e.g. modelling, drawing, words, symbols

Resemblance between the drawing or model and the task scenario described

Relationship between representation and calculation:

Motion e.g. static once created, or involving ongoing movement of elements

Unitariness i.e. whether one representational unit (e.g. cube, tally mark) represents exactly one scenario/numerical unit, or may stand for a group

Spatial structure e.g. grouping of units through separation in space, use of containers, aligning in one or more dimensions

Consistency i.e. whether a single coherent strategy was used from start to finish, or changes occurred

Completeness i.e. whether the external representation had to be 'finished' for solution

Enumeration e.g. unit-counting, step-counting, number fact retrieval

Errors e.g. incorrectly-retrieved number fact, verbal count error

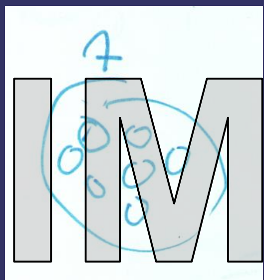
Success i.e. whether the strategy produced a correct solution

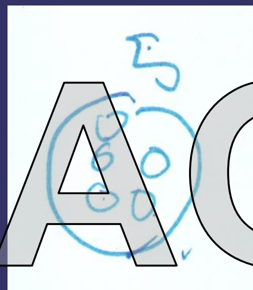
Teacher-student interaction:

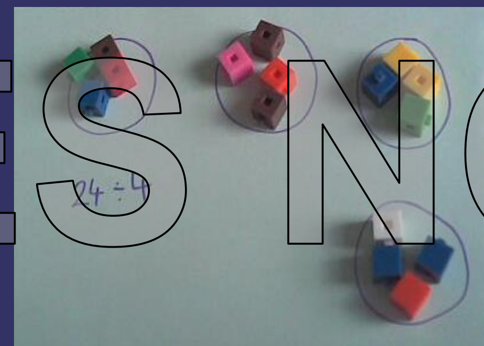
Verbal e.g. spoken prompts, suggestions for calculation

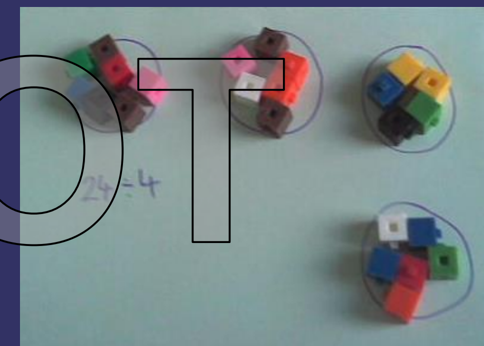
Visuospatial e.g. gestures, participation in modelling/drawing

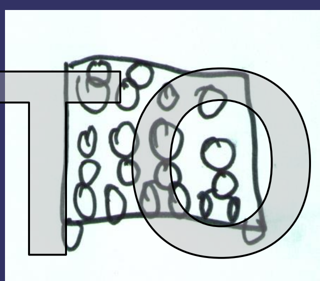
Unit container: *Groups of two or more units enclosed by visible boundaries. Includes representations where units are aligned in rows and/or columns, but these do not represent divisor/quotient or multiplier/multiplicand*

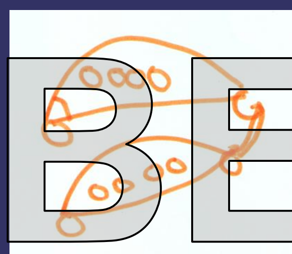


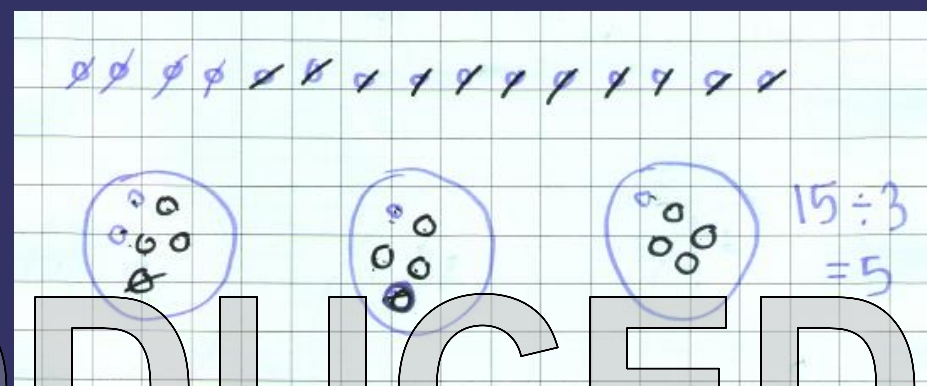


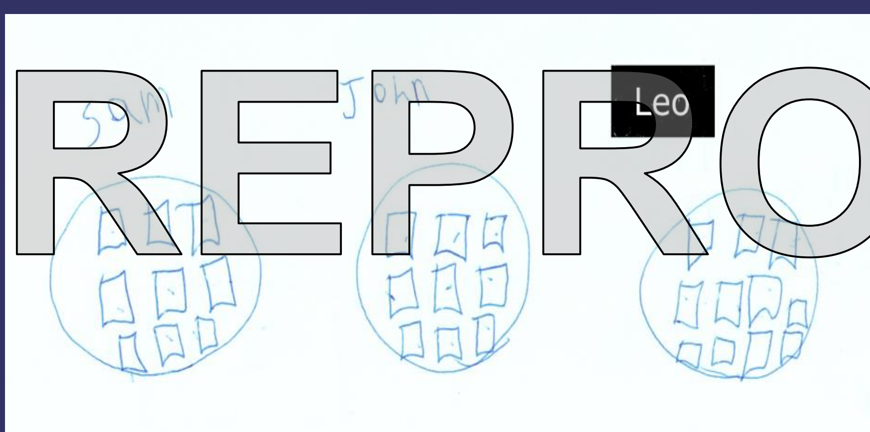












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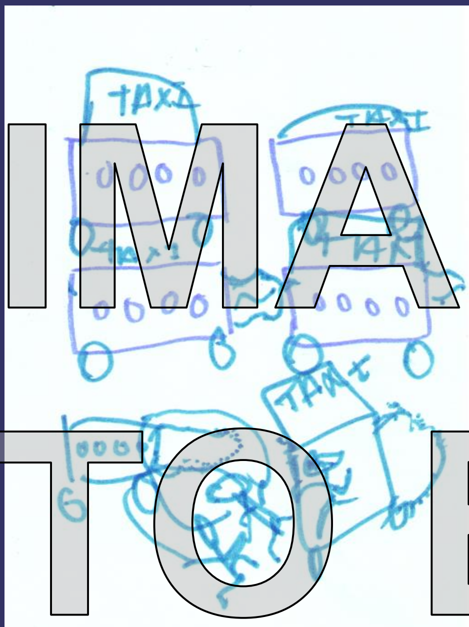
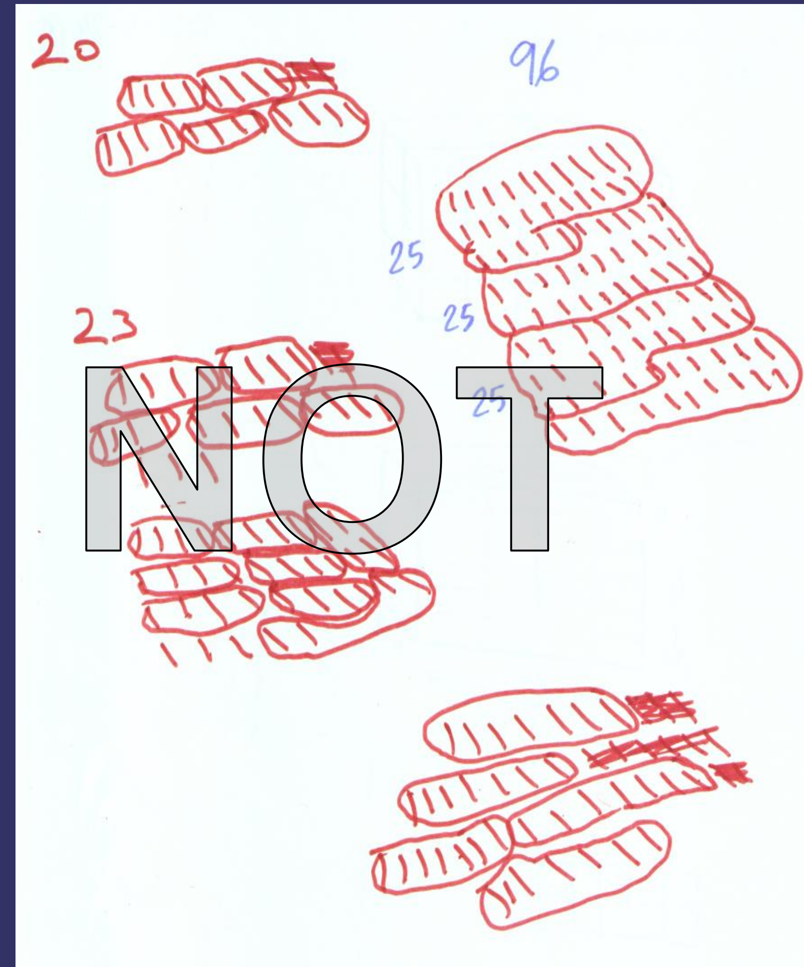
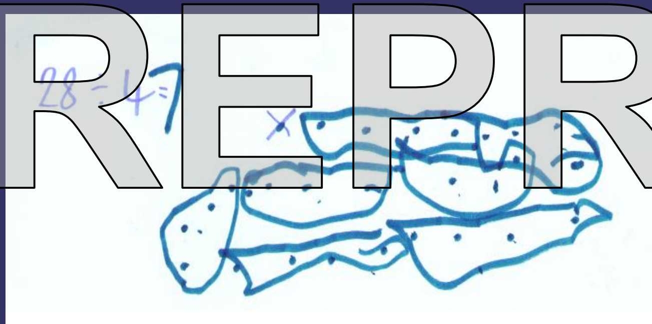
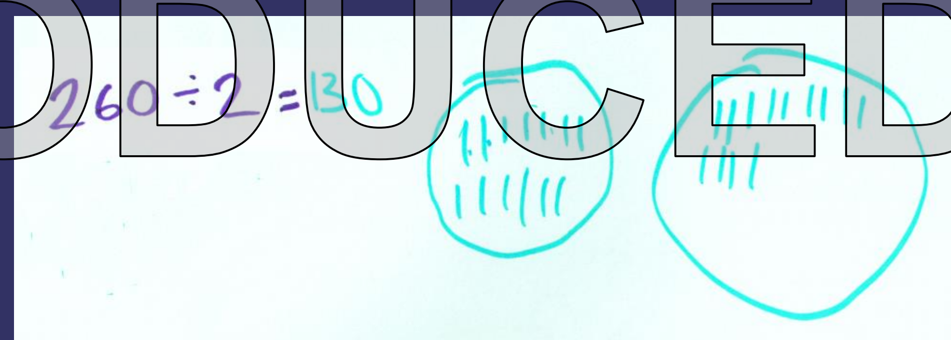
Unit containers (2)









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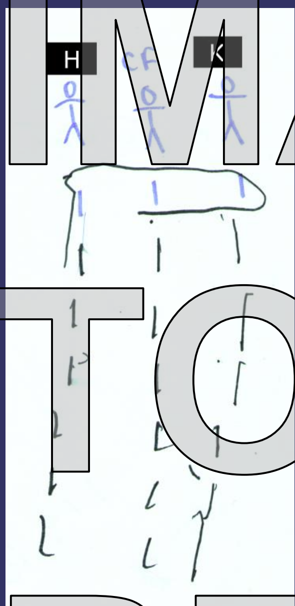
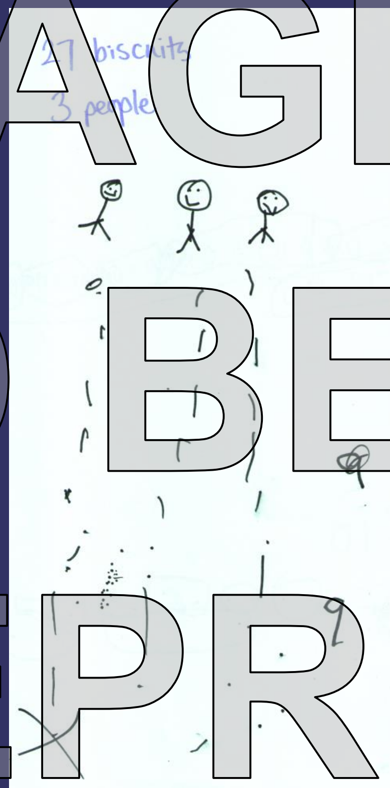
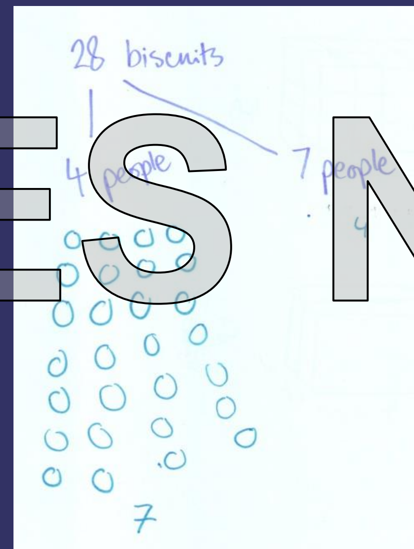
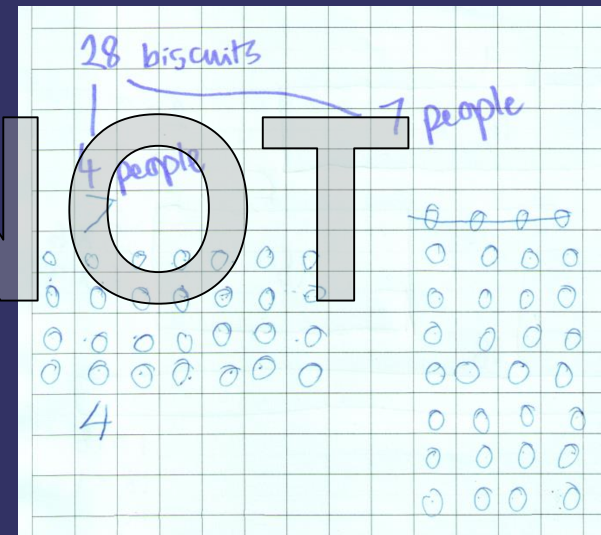
Unit array: *Groups of two or more units aligned in rows and columns, where number of units in the rows/columns represents divisor/quotient or multiplier/multiplicand.*









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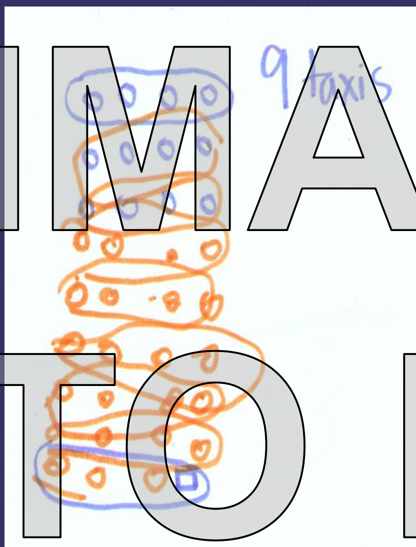
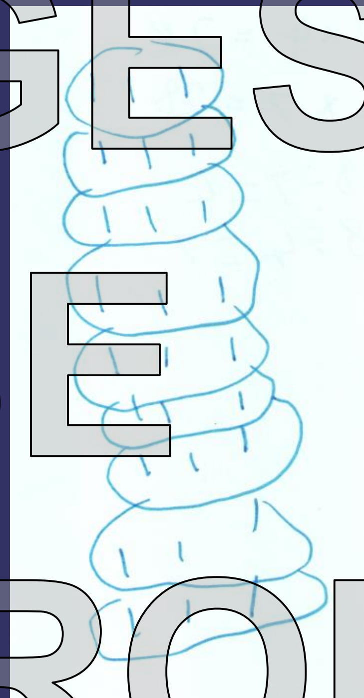
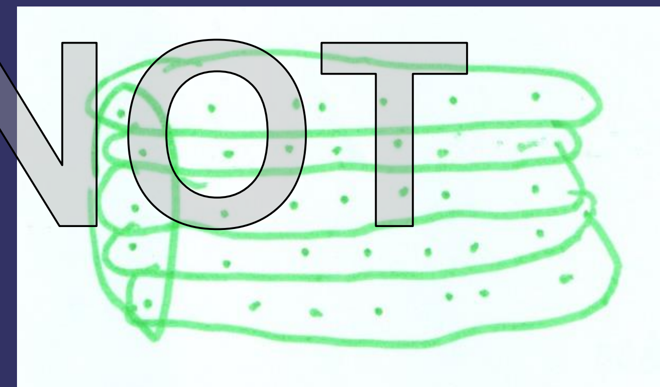
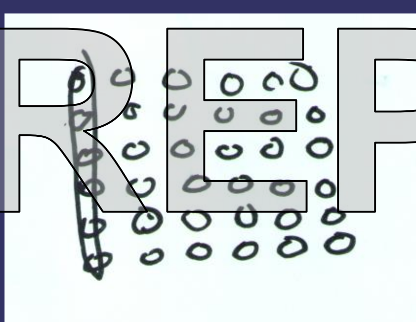
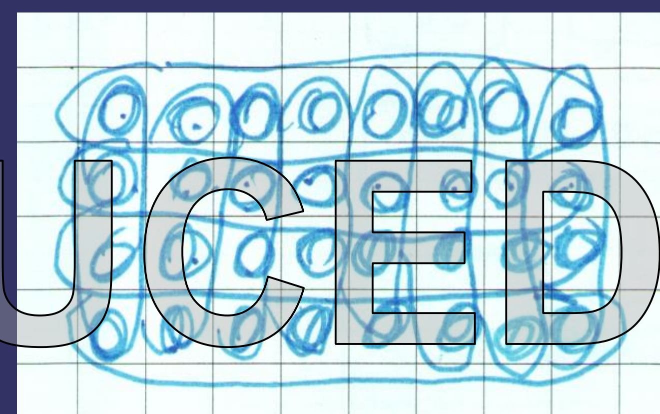
Array-container blend: *Unit array representation with additional containing rings, where number of units in each row/column/container represents divisor/quotient or multiplier/multiplicand.*











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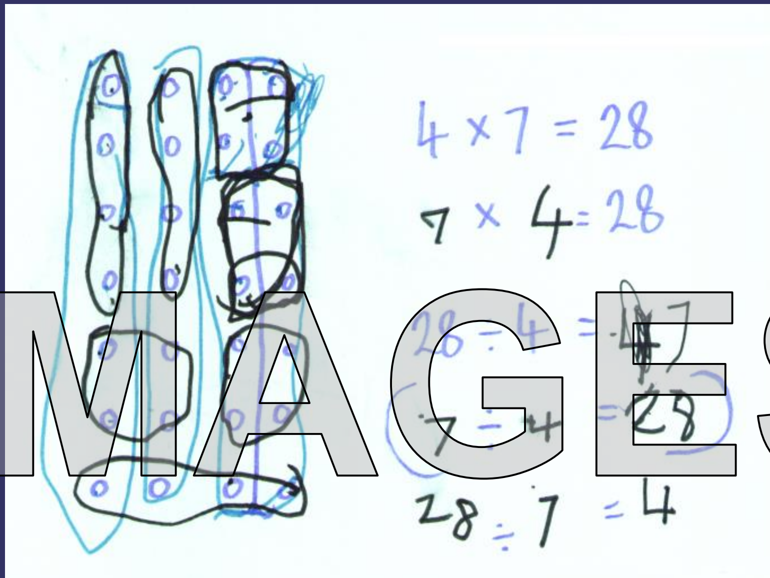
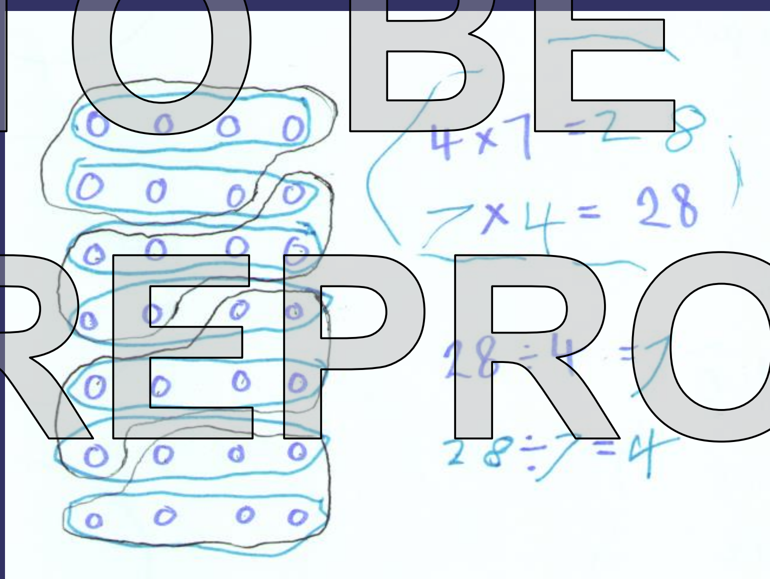
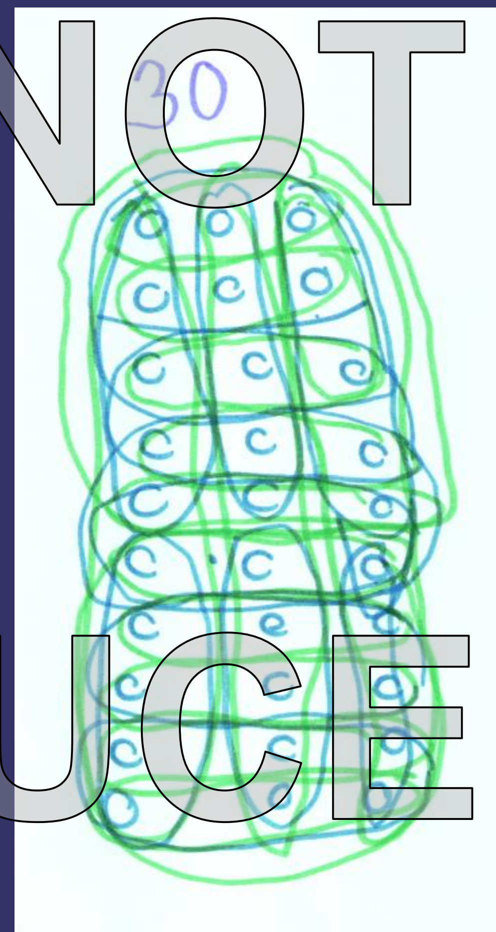
Array-container blends (2)







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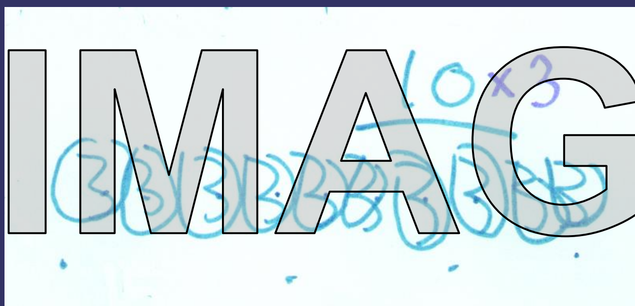
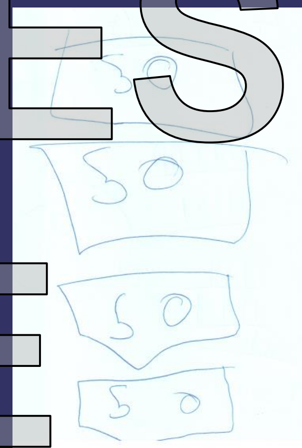
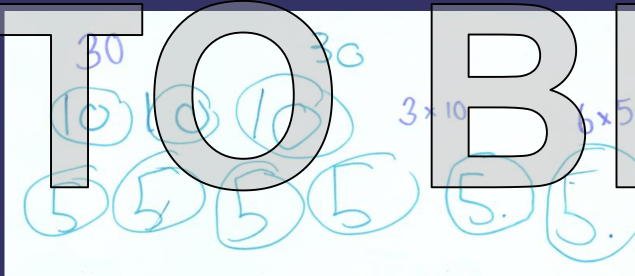
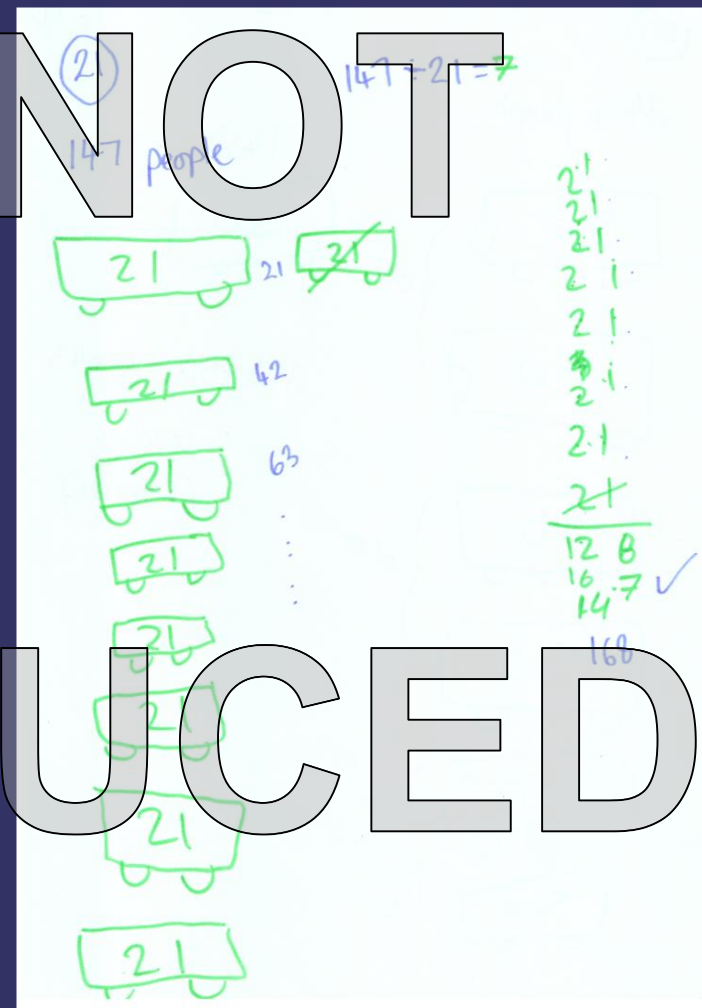
Number container: *Container representation with numerals (rather than unit marks) representing the number in each group written inside, or close by, each container.*









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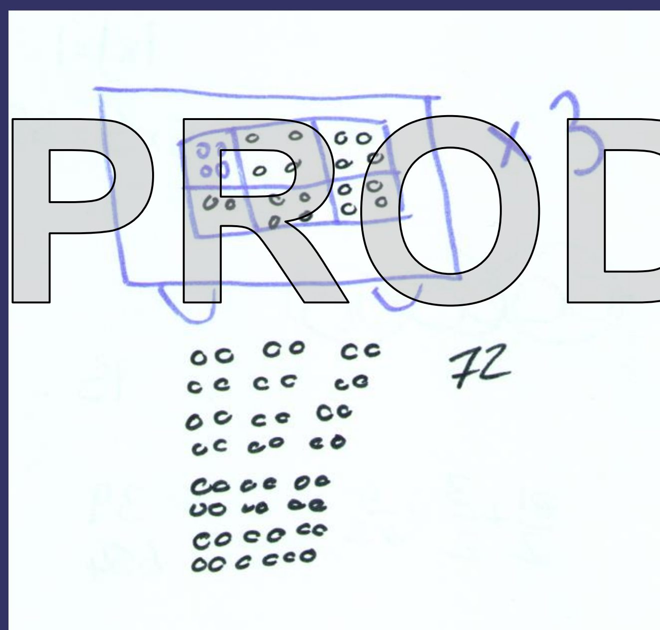
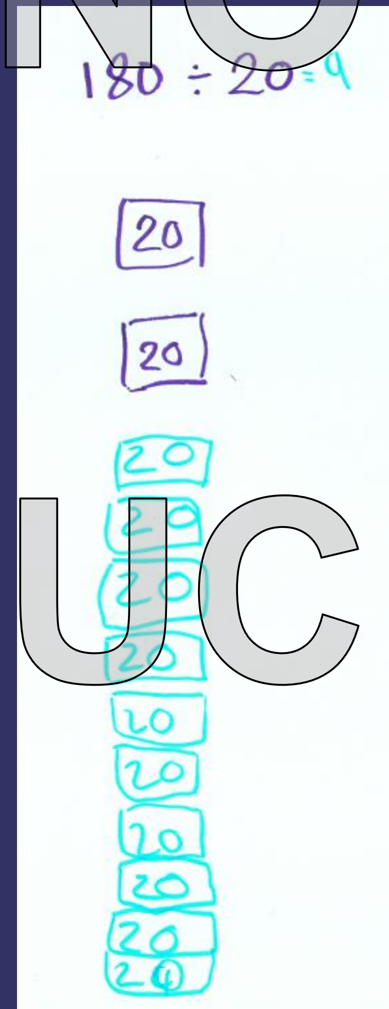
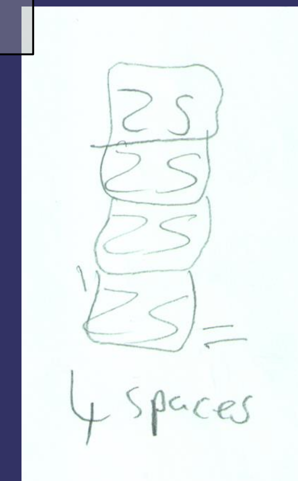
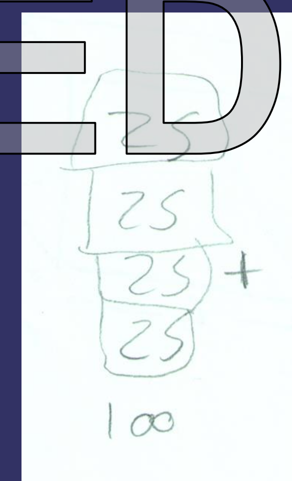
Transitional representations









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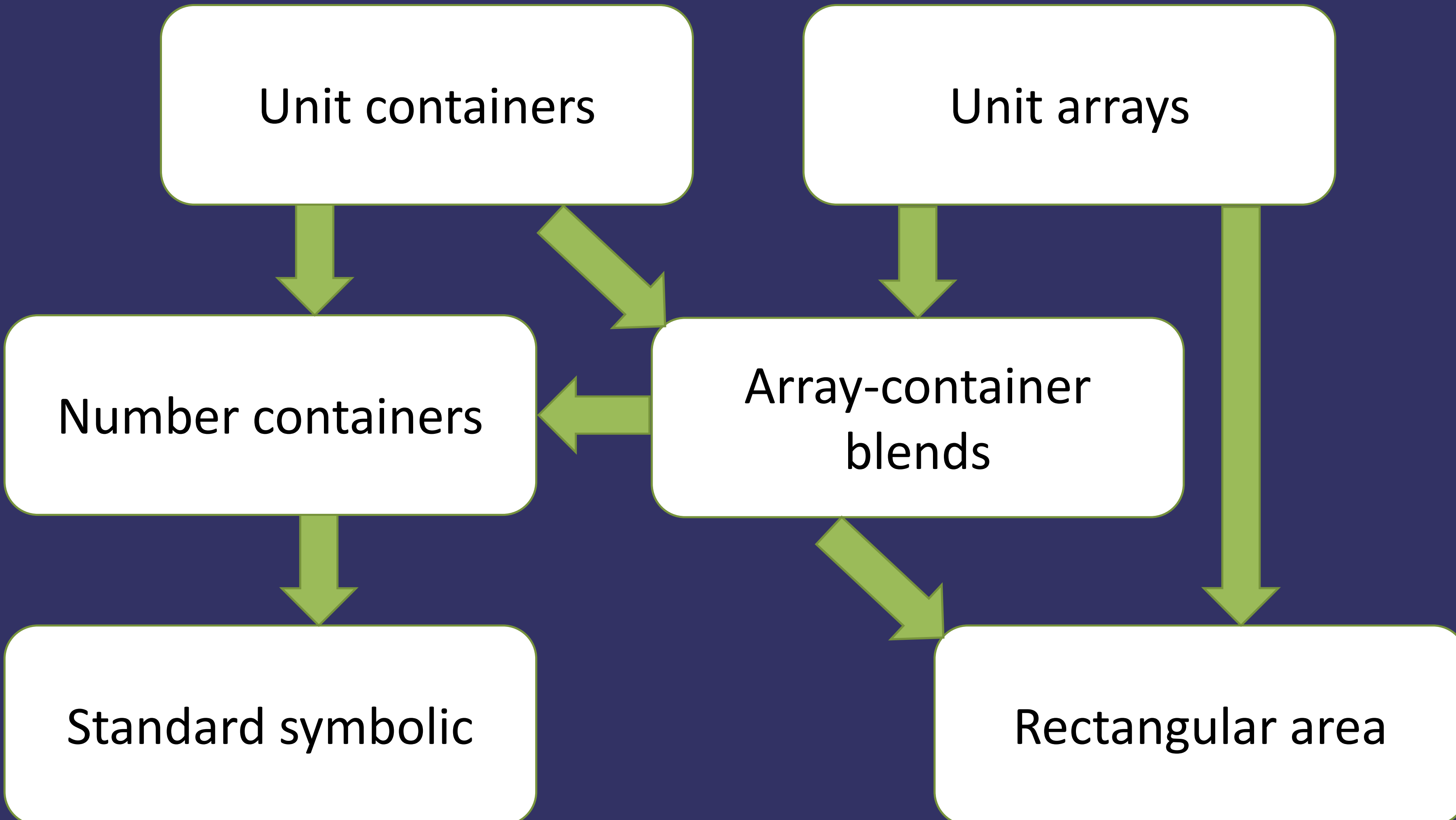
Unit containers
Unit arrays
Number containers
Array-container blends
Standard symbolic
Rectangular area

Representation type	Students' current stage	Subject content	How it helped
Unit containers	Partial or no concept of division	Partitive and quotitive division; relationship with multiplication	Model for sharing and grouping units; seeing repetitive structure
Unit arrays	Partial or procedural concept of division	As above, plus: commutativity of multiplication; multiplicative structures as static relationships	As above, but seeing 2-dimensional repetitive structure
Array-container blends	Using either unit containers or arrays to divide	As above, plus: factorising numbers	As above, plus: seeing equal groups as 'units' in a larger structure
Number containers	Ready to transition from unitary to symbolic representation	Quotitive division; multiplication; recording work	Introducing symbolic notation via familiar imagery

REFERENCES & FURTHER READING

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